

Presented at the Fourth International Wigner Symposium
(Guadalajara, Mexico, August 7 - 11, 1995).

Lorentz Boosts as Squeeze Transformations and Coherence Problems

D. Han

**National Aeronautics and Space Administration,
Goddard Space Flight Center, Code 910.1,
Greenbelt, Maryland 20771, U.S.A.**

Y. S. Kim

**Department of Physics, University of Maryland,
College Park, Maryland 20742, U.S.A.**

Abstract

The quark model and the parton model are known to be two different manifestations of the same covariant entity. However, the interaction amplitudes of partons are incoherent while they are coherent in the quark model. According to Feynman, this is due to the dilation of the interaction time among the quarks. We present a quantitative analysis of this time-dilation problem using Lorentz boosts as squeeze transformations.

Hadrons are believed to be quantum bound states of quarks having localized probability distribution. The most convincing evidence for this bound-state picture is the hadronic mass spectra which are observed in high-energy laboratories [1, 2]. However, this picture of bound states is applicable only to observers in the Lorentz frame in which the hadron is at rest. How would the hadrons appear to observers in other Lorentz frames?

In 1969, Feynman observed that a fast-moving hadron can be regarded as a collection of many “partons” whose properties appear to be quite different from those of quarks [3]. He made the following systematic observations.

- a The picture is valid only for hadrons moving with velocity close to that of light.
- b The interaction time between the quarks becomes dilated, and partons appear to be dilated, and the partons behave as free particles.
- c The momentum distribution of partons becomes widespread as the hadron.
- d The number of partons seems to be infinite or much larger than that of the quarks.

Since Feynman's invention of the parton model, one of the most challenging problems in particle theory has been how to reconcile the above observations with the quark model for hadrons at rest, and there is a model in which the quark model and the parton model are two different manifestation of one covariant entity, as $E = p^2/2m$ and $E = cp$ are two different manifestations of Einstein's $E = mc^2$. [2, 4].

In this report, we would like to discuss in detail item b, which is about one of the basic issues in quantum mechanics. When an external signal interacts with the quarks inside the hadron, we have to make a coherent sum of interaction amplitudes before calculating the total cross section for the hadron. On the other hand, in the parton model, we calculate the cross section for each parton first. The total cross section is therefore an incoherent sum of parton cross sections.

We believe in covariance, and we believe also in quantum mechanics based on the superposition principle. Then does the parton model indicate that Lorentz boosts destroy superposition principle? Indeed, it was Feynman who faced this problem first. His suggestion was that the interaction time between the partons is dilated as the hadron moves fast compared with the time needed for each parton to interact with the external signal. In this report, we give a quantitative analysis of this problem starting from the hadronic wave function at rest, using the squeeze property of the covariant harmonic oscillator formalism.

The Lorentz boost along the z direction is performed according to

$$\begin{pmatrix} z' \\ t' \end{pmatrix} = \begin{pmatrix} \cosh \eta & \sinh \eta \\ \sinh \eta & \cosh \eta \end{pmatrix} \begin{pmatrix} z \\ t \end{pmatrix}, \quad (1)$$

Figure 1: Lorentz-squeezed hadronic distribution. The major axis of this ellipse corresponds to the interaction time among the quarks, while the minor axis measures the time external signal spends inside the hadron.

where η is the boost parameter and is $\tanh^{-1}(v/c)$. In terms of the light-cone variables:

$$u = (z + t)/\sqrt{2}, \quad v = (z - t)/\sqrt{2}. \quad (2)$$

For convenience, we shall call u and v positive and negative light-cone axes respectively. They are perpendicular to each other. The transformation takes a much simpler form:

$$\begin{pmatrix} u' \\ v' \end{pmatrix} = \begin{pmatrix} e^\eta & 0 \\ 0 & e^{-\eta} \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}. \quad (3)$$

If the system is boosted, one axis expands while the other contracts, as is shown in Fig. 1. This boost is clearly a squeeze transformation. [5] The product of the two light-cone variables is

$$uv = (z^2 - t^2)/2, \quad (4)$$

which is a Lorentz-invariant quantity.

Next, let us consider a hadron consisting of two quarks. If the space-time position of two quarks are specified by x_a and x_b respectively, the system can

be described by the variables

$$X = (x_a + x_b)/2, \quad x = (x_a - x_b)/2\sqrt{2}. \quad (5)$$

The four-vector X specifies where the hadron is in space and time, while the variable x measures the space-time separation between the quarks [1]. The simplest wave function in the covariant oscillator system is [2]

$$\psi_0(z, t) = \left(\frac{1}{\pi}\right)^{1/2} \exp\left\{-\frac{1}{2}(z^2 + t^2)\right\}, \quad (6)$$

for the hadron at rest. Quantum excitations along the z direction are allowed, but there are no time-like oscillations along the t direction. In terms of the light-cone variables, the wave function takes the form

$$\psi_0(z, t) = \left(\frac{1}{\pi}\right)^{1/2} \exp\left\{-\frac{1}{2}(u^2 + v^2)\right\}. \quad (7)$$

If the system is boosted, the wave function becomes [2]

$$\psi_\eta(z, t) = \left(\frac{1}{\pi}\right)^{1/2} \exp\left\{-\frac{1}{2}\left(e^{-2\eta}u^2 + e^{2\eta}v^2\right)\right\}. \quad (8)$$

The transition from Eq.(7) to Eq.(8) is a squeeze transformation. The wave function of Eq.(7) is distributed within a circular region in the uv plane, and thus in the zt plane. On the other hand, the wave function of Eq.(8) is distributed in an elliptic region, as in Fig. 1.

As the hadronic speed becomes very close to the speed of light, the elliptic region becomes concentrated along the positive light-cone axis u . The major axis of the ellipse becomes very large, and the minor axis along the v axis becomes very small. They become multiplied by the factors $\exp(\eta)$ and $\eta(-\eta)$ respectively. If the quarks are confined to the thin elliptic region, they move with the speed of light, and the major axis of the ellipse represents the interaction time between the quarks which can now be called partons. Thus the interaction time between the quarks becomes dilated by $\exp(\eta)$.

While the hadron moves along the positive light-cone axis, the interaction signal should come from the opposite direction. It should then come along the negative light-cone axis. The interaction time is approximately the time the signal overlaps with the hadronic distribution. It becomes contracted by

$\exp(-\eta)$. The ratio of the contracted time to the dilated time is therefore $e^{-2\eta}$. The energy of each proton coming out of the Fermilab accelerator is 900GeV . This leads the ratio to 10^{-6} . This is indeed a small number. We know what happened one year ago, but do not know what happened one million years ago. The external signal is not able to sense the interaction of the quarks among themselves inside the hadron.

References

- [1] R. P. Feynman, M. Kislinger, and F. Ravndal, *Phys. Rev. D* **3**, 2706 (1971).
- [2] Y. S. Kim and M. E. Noz, *Theory and Applications of the Poincaré Group* (Reidel, Dordrecht, 1986).
- [3] R. P. Feynman, in *High Energy Collisions*, Proceedings of the Third International Conference, Stony Brook, New York, C. N. Yang *et al.* eds. (Gordon and Breach, New York, 1969); J. D. Bjorken and E. A. Paschos, *Phys. Rev.* **185**, 1975 (1969).
- [4] P. E. Hussar, *Phys. Rev. D* **23**, 2781 (1981); Y. S. Kim, *Phys. Rev. Lett.* **63**, 348-351 (1989); Y. S. Kim, in *Symmetries in Science VIII*, Proceedings, B. Gruber. ed. (Gordon and Breach, 1995).
- [5] D. Han, E. Hardekopf, and Y. S. Kim, *Phys. Rev. A* **39**, 1269 (1989).